HSC 3 March 2017

SECTION - II

Q.4A Multiple choice question (2M each) [6M]

(i) Given that $X \sim B(20, p)$, if $E(X) = 5$ then the value of q is

(a) $\frac{1}{4}$ (b) $\frac{1}{2}$ (c) $\frac{3}{4}$ (d) $\frac{1}{3}$

(ii) If $y = \cos^{-1}\left[\frac{x}{\sqrt{1+x^2}}\right] + \operatorname{cosec}^{-1}\left[\frac{\sqrt{1+x^2}}{x}\right]$, then $\frac{dy}{dx} =$

(a) x (b) 0 (c) 1 (d) $\frac{\pi}{2}$

(iii) $f(x) = x^2 - 3x + 4$ has minimum value at $x =$

(a) 0 (b) $\frac{3}{2}$ (c) $-\frac{3}{2}$ (d) 2

Q.4B Attempt ANY THREE / FIVE (2M each) [6M]

i) Find the approximate value of $\sqrt{25.01}$

ii) $f(x) = \frac{e^{5x} - e^{2x}}{\sin 3x}$ for $x \neq 0$

$= 1$ for $x = 0$ } at $x = 0$

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iii] $y = \sqrt{e^x + \sqrt{e^x + \sqrt{e^x}}}$ find $\frac{dy}{dx}$

iv] Evaluate $\int \sqrt{1+\sin x}\, dx$

v] Form the differential equation whose general solution is

$$x^3 + y^3 = 4ax$$

Q.5[A] Attempt any 2/3 [3M each] [6M]

i) $\int \frac{1}{5+4\sin x}\, dx =$

ii] Verify LMVT for following function

$$f(x) = x + \frac{1}{x}, \quad x \in [1, 3]$$

iii] For the following probability distribution of X

X	0	1	2	3	4
P(X=x)	K	2K	4K	2K	K

find :- i] K ii] P(X>0) iii] P(X≤1)

B] Attempt any 2/3 (4M each) [8M]

i] If $ax^2+2hxy+by^2=0$ then show that

$$\frac{d^2y}{dx^2}=0$$

ii] Examine continuity of the function $f(x)$ at $x=0$, where

$$f(x)=\frac{10^x+7^x-14^x-5^x}{1-\cos 4x}, \text{ for } x\neq 0$$

$$=\frac{10}{7} \text{ , for } x=0$$

iii] Evaluate $\int_0^{\pi/4} \log(1+\tan x)\,dx$

Q.6(A) Attempt any 2/3 (3M each) [6M]

i] If u and v are two functions of x then prove that

$$\int uv\,dx = u\int v\,dx - \int\left[\frac{du}{dx}\int v\,dx\right]dx$$

ii] If $y=f(u)$ is a differentiable function of u and $u=g(x)$ is a differentiable function of x, then prove that $y=f[g(x)]$ is a differentiable function of x and

$$\frac{dy}{dx}=\frac{dy}{du}\cdot\frac{du}{dx}$$

iii) Let the pmf of r.v. X be

$$P(x) = \binom{4}{x}\left(\frac{1}{2}\right)^{x}\left(\frac{1}{2}\right)^{4-x}, \quad x = 0, 1, 2, 3, 4$$

Find E(x) and Var(x)

B] Attempt any 2/3 (4M each) [8M]

i] Following is the distribution function F(x) of a discrete r.v. X

x	1	2	3	4	5	6
F(x)	0.2	0.37	0.48	0.62	0.85	1

i) Find the probability distribution of X

ii) Find $P(X \leq 3)$, $P(2 < X < 5)$

iii) Find $P(X \leq 5 \mid X > 3)$

ii) Prove that $\int_{-a}^{a} f(x)\,dx = 2\int_{0}^{a} f(x)\,dx$ (if f(x) is an even function)

$= 0$, if f(x) is an odd f(x)

iii) If $y = \sin(m \sin^{-1} x)$, show that

$$(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} + m^2 y = 0$$