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Solution

Section-II

Q.4) A) M.C.Q

Q.i) We know $X \sim B(n,p)$

$\therefore n = 20$ is given

~~Given $E(x) = np$~~

Given $E(x) = 5$

We know $E(x) = np$

$\therefore np = 5$

$\therefore p = \frac{5}{n} = \frac{5}{20} = 1/4$

$p + q = 1$, $\therefore q = 1 - p$

$\therefore q = 1 - 1/4 = \frac{3}{4}$

(c) 3/4

Q.ii) We know

$\sin^{-1}x + \cos^{-1}x = \frac{\pi}{2}$ — (1)

$y = \cos^{-1}\left[\frac{x}{\sqrt{1+x^2}}\right] + \operatorname{cosec}^{-1}\left[\frac{\sqrt{1+x^2}}{x}\right]$

$\therefore y = \cos^{-1}\left[\frac{x}{\sqrt{1+x^2}}\right] + \sin^{-1}\left[\frac{x}{\sqrt{1+x^2}}\right]$

$\therefore y = \pi/2$ — from (1)

Differentiate w.r.t x we get

$\frac{dy}{dx} = \frac{d(\pi/2)}{dx} = 0$

$\left[\because \frac{\pi}{2} \text{ is constant}\right]$

Q.iii) $f(x) = x^2 - 3x + 4$

To find minimum value

i) Differentiate $f(x)$ wrt x

i.e $f'(x)$

(ii) Put $f'(x) = 0$

From (ii) eq^n you will get 1, 2 or 3 values of x

(iii) Take double derivative of $f'(x)$

i.e differentiate $f'(x)$ again (Double derivative)

(iv) Check for which values of x [obtained from eq^n (ii)] $f''(x) > 0$ (For finding minimum value of a function)

In this question

$f(x) = x^2 - 3x + 4$

$f'(x) = 2x - 3$ — 1

Put $f'(x) = 0$

$\therefore 2x - 3 = 0$

$\therefore x = 3/2$

Differentiating eqn 1 again

$f''(x) = 2$

i.e at $x = 3/2$

$f''(\frac{3}{2}) = 2$

i.e $f''(\frac{3}{2}) > 0$

$\therefore$ Function has min. value at $x = 3/2$

(b) $\frac{3}{2}$

Q.4) B)

i) $f(x) = \sqrt{x}$ — 1

To find $\sqrt{25.01}$

i.e $x = 25.01$

Let $a = 25$

$h = +0.01$

$f(a+h) \doteq f(a) + hf'(a)$

To find $f(a+h)$ we must know

$\underline{f(a)} \rightarrow$ Calc this by

Substituting $x = a = 25$ in eqn 1

$\therefore f(a) = f(25) = \sqrt{25} = 5$

For finding $f'(a)$

Diff eqn 1 wrt x

$f'(x) = \frac{1}{2\sqrt{x}}$

$f'(a) = \frac{1}{2\sqrt{25}} = \frac{1}{2 \times 5}$

$\therefore f'(a) = \frac{1}{10} = 0.1$

$\therefore f(a+h) \doteq 5 + 0.01 \times 0.1$

$\doteq 5 + 0.001$

$\therefore f(25 + 0.01) \doteq 5.001$

$\boxed{\therefore f(25.01) \doteq 5.001}$

Q.4)B) ii) $f(x) = \dfrac{e^{5x} - e^{2x}}{\sin 3x}, \quad x \neq 0$

$= 1 \quad , x = 0$

TO CHECK WHETHER THE GIVEN FUNCTION IS CONTINUOUS OR NOT

$$\lim_{x\to 0} f(x) = \lim_{x\to 0} \frac{e^{5x} - e^{2x}}{\sin 3x}$$

$\frac{0}{0}$ Nr & Dr by x

$$= \lim_{x\to 0} \frac{\frac{e^{5x} - e^{2x}}{x}}{\frac{\sin 3x}{x}}$$

$$= \lim_{x\to 0} (e^{2x}) \frac{\left[\frac{e^{3x}-1}{3x}\right]}{\frac{\sin 3x}{3x}}$$

$$= \frac{\lim_{x\to 0} e^{2x} \times \lim_{x\to 0} \frac{e^{3x}-1}{3x}}{\lim_{x\to 0} \frac{\sin 3x}{3x}}$$

$$= 1 \times \frac{1}{1} = 1$$

$\left(\because \lim_{x\to 0} \frac{e^x - 1}{x} = 1, \lim_{x\to 0} \frac{\sin x}{x} = 1\right)$

Q. iii) $y = \sqrt{e^x + \sqrt{e^x + \sqrt{e^x}}}$

Differentiating wrt x

[Note :- This quest^n is easy only you need to take care of opening & closing brackets. That's why while solving Maths Paper Be Calm & Don't Take Tension Solve Paper Slowly]

$$\frac{dy}{dx} = \frac{d}{dx}\left(\sqrt{e^x + \sqrt{e^x + \sqrt{e^x}}}\right)$$

$$= \frac{1}{2\sqrt{e^x + \sqrt{e^x + \sqrt{e^x}}}} \times \frac{d(e^x + \sqrt{e^x + \sqrt{e^x}})}{dx}$$

$$= \frac{1}{2y}\left[\frac{d}{dx}e^x + \frac{d(\sqrt{e^x + \sqrt{e^x}})}{dx}\right]$$

$$= \frac{1}{2y}\left[e^x + \frac{1}{2\sqrt{e^x + \sqrt{e^x}}} \frac{d(e^x + \sqrt{e^x})}{dx}\right]$$

$$= \frac{1}{2y}\left[e^x + \frac{1}{2\sqrt{e^x + \sqrt{e^x}}}\left[e^x + \frac{1}{2\sqrt{e^x}}\right]\right]$$

$$= \frac{1}{2y}\left[e^x + \frac{1}{2\sqrt{e^x + \sqrt{e^x}}} \frac{[2e^x\sqrt{e^x} + 1]}{\times 2\sqrt{e^x}}\right]$$

$$= \frac{1}{2y}\left[e^x + \frac{2e^x\sqrt{e^x} + 1}{4\sqrt{e^x + \sqrt{e^x}}\sqrt{e^x}}\right]$$

Q.4 B) iv) $\int \sqrt{1+\sin x}\, dx$

We know,

$\left(\sin\frac{x}{2} + \cos\frac{x}{2}\right)^2 =$

$\sin^2\frac{x}{2} + \cos^2\frac{x}{2} + 2\sin\frac{x}{2}\cos\frac{x}{2} = 1 + \sin x$

~~1 + sin x~~

Let $I = \int \sqrt{1+\sin x}\, dx$

$= \int \sqrt{\left(\sin\frac{x}{2} + \cos\frac{x}{2}\right)^2}\, dx$

$= \int \left[\sin\frac{x}{2} + \cos\frac{x}{2}\right] dx$

$= \dfrac{-\cos\frac{x}{2}}{\frac{1}{2}} + \dfrac{\sin\frac{x}{2}}{\frac{1}{2}} + C$

(bcz $\frac{x}{2}$) ←

$= -2\cos\frac{x}{2} + 2\sin\frac{x}{2} + C$

Q.v] Since there is 1 constant therefore diff it one time

$3x^2 + 3y^2\frac{dy}{dx} = 4a$ → [Remove this]

$\therefore 3x^2 + 3y^2\frac{dy}{dx} = 4\left[\frac{x^3+y^3}{4x}\right] = \frac{x^3+y^3}{x}$

$\therefore 3y^2\frac{dy}{dx} + \underline{3x^2 - x^2} - \frac{y^3}{x} = 0$

Q.5.A] Let $I = \int \frac{1}{5+4\sin x} dx$

It is of the type

$\int \frac{1}{a\sin x + b\cos x + c} dx$, where $b=0$

For such type of questⁿ put $\tan\frac{x}{2} = t$

Replace $\sin x = \frac{2t}{1-t^2}$

Ans] $\int \frac{2}{5-5t^2+8t} dt$

$= \frac{2}{-5}\int \frac{1}{t^2 - \frac{8}{5}t - 1} dt$

$= \frac{2}{-5}\int \frac{1}{\left(t-\frac{4}{5}\right)^2 - 1 - \frac{16}{25}} dt$

$= \frac{-2}{5}\int \frac{1}{\left(t-\frac{4}{5}\right)^2 - \frac{41}{25}} dt$

$= \frac{-\cancel{2}}{\cancel{5}} \frac{1}{\frac{\cancel{2}\sqrt{41}}{\cancel{5}}} \log\left|\frac{t-\frac{4}{5}-\sqrt{\frac{41}{25}}}{t-\frac{4}{5}+\sqrt{\frac{41}{25}}}\right| + c$

$= -\frac{1}{\sqrt{41}} \log\left|\frac{5t-4-\sqrt{41}}{5t-4+\sqrt{41}}\right| + c$

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Q.5-A.ii) Whenever LMVT / Rolle's Theorem is asked don't forget to write 1st & 2nd condition satisfied

Since the above given fⁿ is polynomial therefore it is continuous on its domain i.e $[1,3]$ [Note [ ] → bracket]

Also it is differentiable for $x \in (1,3)$ [Note ( ) → bracket]

Therefore all the condition of LMVT are satisfied

To find c such that it lies between $(1,3)$

$f'(c) = \frac{f(b)-f(a)}{b-a} = \frac{f(3)-f(1)}{3-1}$

$1 - \frac{1}{x^2} = \frac{3+\frac{1}{3} - (1+1)}{2}$

$1 - \frac{1}{x^2} = \frac{3+\frac{1}{3}-2}{2} = \frac{1+\frac{1}{3}}{2}$

$1 - \frac{1}{x^2} = \frac{4}{3(2)} = \frac{2}{3}$

$\therefore \frac{1}{x^2} = 1 - \frac{2}{3} = \frac{1}{3}$

$\therefore x^2 = 3,\ x = \pm\sqrt{3}$

$-\sqrt{3} \notin$ to $(1,3)$

$\therefore c = \sqrt{3}$ which lies $(1,3)$

Q.III] i) $\Sigma P(x) = 1$

$k + 2k + 4k + 2k + k = 1$

$10k = 1$

$k = \frac{1}{10}$

ii) $P(x > 0) = 1 - P(x \leq 0)$

$= 1 - P(x = 0)$

$= 1 - k = 1 - \frac{1}{10} = \frac{9}{10}$

iii) $P(x \leq 1) = P(x=0) + P(x=1)$

$= k + 2k = 3k = \frac{3}{10}$

Q.5.B) i) $ax^2 + 2hxy + by^2 = 0$ — (1)

Diff wrt x we get

$2ax + 2h\left[x\frac{dy}{dx} + y\right] + 2by\frac{dy}{dx} = 0$

$2ax + 2hx\frac{dy}{dx} + 2hy + 2by\frac{dy}{dx} = 0$

$2hx\frac{dy}{dx} + 2by\frac{dy}{dx} = -[2ax + 2hy]$

$\frac{dy}{dx}[2hx + 2by] = -2[ax + hy]$

$\therefore \frac{dy}{dx} = \frac{-2[ax+hy]}{2[hx+by]}$

$\therefore \frac{dy}{dx} = -\frac{(ax+hy)}{hx+by}$ — (2)

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Next Step is more important

Now don't diff again because sum will become tedious

Eq^n (1) can be rewritten as

$ax^2 + hxy + hxy + by^2 = 0$

$x(ax + hy) + y(hx + by) = 0$

$-x(ax+hy) = +y(hx+by)$

$y = -x\frac{(ax+hy)}{hx+by}$

$\boxed{\therefore \frac{y}{x} = -\left(\frac{ax+hy}{hx+by}\right)}$

Substitute in eq^n (2)

$\frac{dy}{dx} = \frac{y}{x}$, then diff

$\frac{d^2y}{dx^2} = \frac{x\frac{dy}{dx} - y}{x^2}$

$\therefore \frac{d^2y}{dx^2} = \frac{x\left(\frac{y}{x}\right) - y}{x^2}$ $\left[\because \frac{dy}{dx} = \frac{y}{x}\right]$

$\therefore \frac{d^2y}{dx^2} = \frac{y - y}{x^2} = 0$

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Q.5.B)-ii) $\lim_{x\to 0} f(x) = \lim_{x\to 0} \frac{10^x - 5^x + 7^x - 14^x}{1-\cos 4x}$

$= \lim_{x\to 0} \frac{5^x[2^x - 1] + 7^x[1-2^x]}{2\sin^2 2x}$

$\div$ Nr & Dr by x^2

$= \lim_{x\to 0} \frac{\frac{(5^x - 7^x)(2^x-1)}{x^2}}{\frac{2\sin^2 2x}{x^2}} = \frac{1}{2}\lim_{x\to 0} \frac{\left(\frac{5^x - 7^x}{x}\right)\left(\frac{2^x-1}{x}\right)}{(2)^2}$

$= \frac{1}{2\times 4} \lim_{x\to 0}\left[\frac{5^x - 1 - (7^x - 1)}{x}\right] \log 2$

$= \frac{1}{8}(\log 5 - \log 7)\log 2 = \frac{1}{8}\log\frac{5}{7}\times \log 2$

$\because \lim_{x\to 0} f(x) \neq f(0)$

$\therefore f^n$ is discontinuous at $x = 0$

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Q. 1ii) $\int_0^{\pi/4} \log(1+\tan x)dx = \int_0^{\pi/4} \log\left[1+\tan\left(\frac{\pi}{4}-x\right)\right]dx$

$\left[\because \int_0^a f(x)dx = \int_0^a f(a-x)dx\right]$

$= \int_0^{\pi/4} \log\left[1 + \frac{1-\tan x}{1+\tan x}\right]dx = \int_0^{\pi/4} \log\left[\frac{1+\tan x + 1 - \tan x}{1+\tan x}\right]dx$

$= \int_0^{\pi/4} \log\frac{2}{1+\tan x}dx = \int_0^{\pi/4} [\log 2 - \log(1+\tan x)]dx$

$\therefore I = \log 2\int_0^{\pi/4} dx - \underbrace{\int_0^{\pi/4} \log(1+\tan x)dx}_{I}$

$2I = \log 2 \, [x]_0^{\pi/4} = \frac{\pi}{4} \log 2$

$$\boxed{\therefore I = \frac{\pi}{8} \log 2}$$

Q.(A) i) $\frac{d}{dx} u \cdot w = u \frac{dw}{dx} + w \frac{du}{dx}$ — ①

Put $w = \int v\,dx$, $\frac{dw}{dx} = v$

~~Integrate~~ eqⁿ ① ~~we get~~ becomes

$\frac{d}{dx} u \cdot w = uv + \frac{du}{dx} \int v\,dx$

Integrate above eqⁿ wrt x

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$u \cdot w = \int uv\,dx + \int \left(\frac{du}{dx} \int v\,dx\right) dx$

$\therefore \int uv\,dx = uw - \int \left(\frac{du}{dx} \int v\,dx\right) dx$

$$\boxed{\int uv\,dx = u \int v\,dx - \int \left(\frac{du}{dx} \int v\,dx\right) dx}$$

ii) To prove any theorem of derivative write in 3 steps

i) let there be small increment in x

ii) Write conditions for differentiable f^n

iii) whatever you want to prove write in δ notation then take limit

Convert δ into d using (2) eqⁿ.

Q.6(A·ii) (1) let δx be a small increment in x, let δy & δu be corresponding increment in y & u respectively

(2) It is given that y is a differentiable f^n of u

$\therefore \frac{dy}{du} = \lim_{\delta u \to 0} \frac{\delta y}{\delta u}$ exist and is finite

Also u is a differentiable f^n of x

$\therefore \frac{du}{dx} = \lim_{\delta x \to 0} \frac{\delta u}{\delta x}$ exist and is finite

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(3) $\frac{\delta y}{\delta x} = \frac{\delta y}{\delta u} \times \frac{\delta u}{\delta x}$

Take limit $\delta x \to 0$ on both sides

$$\lim_{\delta x \to 0} \frac{\delta y}{\delta x} = \lim_{\delta x \to 0} \left(\frac{\delta y}{\delta u} \times \frac{\delta u}{\delta x} \right)$$

$$\therefore \lim_{\delta x \to 0} \frac{\delta y}{\delta x} = \lim_{\delta x \to 0} \frac{\delta y}{\delta u} \times \lim_{\delta x \to 0} \frac{\delta u}{\delta x} = \lim_{\delta u \to 0} \frac{\delta y}{\delta u} \times \lim_{\delta x \to 0} \frac{\delta u}{\delta x}$$

$$\lim_{\delta x \to 0} \frac{\delta y}{\delta x} = \frac{dy}{du} \times \frac{du}{dx} \quad \text{— (I)}$$

Since R.H.S of eq(I) exists & is finite

$\therefore \lim_{\delta x \to 0} \frac{\delta y}{\delta x}$ also exists & finite

$$\therefore \boxed{\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}}$$

Q.(ii) $\binom{n}{r}$ → This symbol denotes ${}^{n}C_{r}$

$P(0) = {}^{4}C_{0}\left(\frac{1}{2}\right)^{0}\left(\frac{1}{2}\right)^{4} = 1 \times \frac{1}{2^4}$

$P(1) = {}^{4}C_{1}\left(\frac{1}{2}\right)^{1}\left(\frac{1}{2}\right)^{3} = \frac{4}{2^4} = \frac{4}{2^4}$

$P(2) = {}^{4}C_{2}\left(\frac{1}{2}\right)^{2}\left(\frac{1}{2}\right)^{2} = \frac{4\cdot 3}{1\cdot 2} \times \frac{1}{2^4} = \frac{6}{2^4}$

$P(3) = {}^{4}C_{3}\left(\frac{1}{2}\right)^{3}\left(\frac{1}{2}\right)^{1} = \frac{4}{2^4}$

$P(4) = {}^{4}C_{4}\left(\frac{1}{2}\right)^{4}\left(\frac{1}{2}\right)^{0} = \frac{1}{2^4}$

X	0	1	2	3	4
P(X)	$\frac{1}{2^4}$	$\frac{4}{2^4}$	$\frac{6}{2^4}$	$\frac{4}{2^4}$	$\frac{1}{2^4}$

$E(x) = \sum x P(x) = 0 + \frac{4}{2^4} + \frac{2(6)}{2^4} + \frac{3(4)}{2^4} + \frac{4}{2^4}$

$= \frac{32}{2^4} = \frac{32}{16} = 2$

$Var(x) = E(x^2) - [E(x)]^2$

$E(x^2) = \sum x^2 P(x) = 0 + \frac{4}{2^4} + \frac{24}{2^4} + \frac{36}{2^4} + \frac{16}{2^4}$

$= \frac{80}{16} = \frac{10}{2} = 5$

$Var(x) = 5 - 4 = 1$

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Q.6 B) i) i)

X	1	2	3	4	5	6
F(X)	0.2	0.37	0.48	0.62	0.85	1
P(X)	0.2	0.17	0.11	0.14	0.23	0.15

probability distribution

for find P(X) subtract two consecutive value of F(x)

ii) $P(X \le 3) = F(3) = 0.48$

$P(2 < x < 5) = P(3) + P(4) = 0.25$

iii) $P(X \le 5 \mid X > 3) = P(X=4) + P(X=5)$

$= 0.14 + 0.23 = 0.37$

ii) $\int_{-a}^{a} f(x)\,dx \rightarrow \int_{-a}^{0} f(x)\,dx + \int_{0}^{a} f(x)\,dx$

$\int_{-a}^{0} f(x)\,dx$, Put $x = -t$

x	-a	0
t	a	0

$\int_{a}^{0} f(-t) -dt = -\int_{a}^{0} f(-t)\,dt = \int_{0}^{a} f(-t)\,dt$

$= \int_{0}^{a} f(-x)\,dx$

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$\therefore \int_{-a}^{a} f(x)\,dx = \int_{0}^{a} f(-x)\,dx + \int_{0}^{a} f(x)\,dx$

When $f(-x) = -f(x)$ [$f(x) \rightarrow$ odd]

$\int_{-a}^{a} f(x)\,dx = -\int_{0}^{a} f(x)\,dx + \int_{0}^{a} f(x)\,dx = 0$

When $f(-x) = f(x)$ [$f(x) \rightarrow$ even]

$\int_{-a}^{a} f(x)\,dx = 2\int_{0}^{a} f(x)\,dx$

X.6.B (iii) $y = \sin(m\sin^{-1}x)$

Diff wrt x we get

$$\frac{dy}{dx} = \cos(m\sin^{-1}x)\frac{m}{\sqrt{1-x^2}}$$

$$\sqrt{1-x^2}\frac{dy}{dx} = m\cos(m\sin^{-1}x)$$

Diff once again

$$\sqrt{1-x^2}\frac{d^2y}{dx^2} + \frac{dy}{dx}\frac{1}{2\sqrt{1-x^2}}(-2x) = -m\sin(m\sin^{-1}x)\times\frac{m}{\sqrt{1-x^2}}$$

Multiply above eq^n with $\sqrt{1-x^2}$ we get

$$(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} = -m^2\sin(m\sin^{-1}x)$$

y

$$\therefore (1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} + m^2y = 0$$