HSC 3 March 2017

Time: 3 Hrs Max. Mks: 80

Section - I

Q.1) A) Multiple Choice Question (2M each) [6M]

i) If a line makes an angle 45° with Y and Z axis, then find the angle which it makes with X-axis.

(a) $\frac{\pi}{3}$ (b) $\frac{\pi}{6}$ (c) $\frac{\pi}{4}$ (d) $\frac{\pi}{2}$

ii) Find direction cosines of the line passing through the points A(2, 3, 4) & B(1, 0, −2)

(a) $\frac{\pm 1}{\sqrt{5}}, \frac{\pm 3}{\sqrt{5}}, \frac{\mp 6}{\sqrt{5}}$

(b) $\frac{\pm 1}{\sqrt{40}}, \frac{\pm 3}{\sqrt{40}}, \frac{\pm 6}{\sqrt{40}}$

(c) $\frac{\pm -1}{\sqrt{46}}, \frac{\pm -3}{\sqrt{46}}, \frac{\pm -6}{\sqrt{46}}$

(d) $\pm \frac{1}{\sqrt{46}}, \mp \frac{3}{\sqrt{46}}, \mp \frac{6}{\sqrt{46}}$

iii) Find value of q, if the equation

$$qx^2 + 4xy + 4y^2 + ax + by + c = 0$$

represents a pair of parallel lines.

(a) -1 (b) 2 (c) -4 (d) 1

B) Attempt any 3/5

i) Show that the points $(1,-1,3)$ and $(3,4,3)$ are equidistance from the plane

$$\bar{r} \cdot (5\bar{i} + 2\bar{j} - 7\bar{k}) = -8$$

ii) Write the converse and inverse of the following statement

"If surface area decreases then the pressure increases"

iii) If $A = \begin{bmatrix} 2 & -2 \\ 4 & 3 \end{bmatrix}$ then find A^{-1} by the adjoint method

iv) Find the Cartesian co-ordinates of the points whose polar coordinates are $(4, \frac{\pi}{2})$

v) Find the vector equation of the line passing through $A(3,4,-7)$ and $B(1,2,3)$. Also find the cartesian form.

Q.2A) Attempt any 2/3 (3M each) [6M]

i) By vector method prove that the angle subtended on a semicircle is a right angle.

ii) Without using truth tables, show that

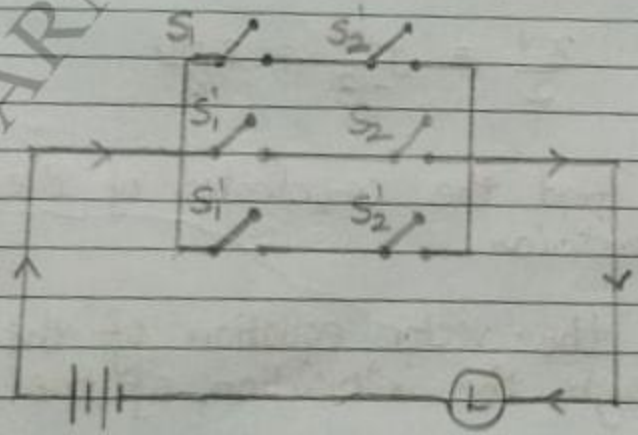
$$p \leftrightarrow q \equiv (p \wedge q) \vee (\sim p \wedge \sim q)$$

iii) In ΔABC, show that

$$\frac{\cos^2 B - \cos^2 C}{b+c} + \frac{\cos^2 C - \cos^2 A}{c+a} = \frac{\cos^2 A - \cos^2 B}{a+b}$$

Q.2B) Attempt any 2/3 [4M each) [8M]

i) Give an alternative arrangement for the following circuit, so that the new circuit has 2 switches only. Also write the switching table



ii) Find the inverse of $\begin{bmatrix} 1 & 2 & 3 \\ 1 & 1 & 5 \\ 2 & 4 & 7 \end{bmatrix}$ by using elementary row transformation.

iii) Find the general solution of

$$\sin x \tan x - \tan x + \sin x - 1 = 0$$

Q.3) Attempt any 2/3 (3M each) [6M]

i) If the points A(3, 0, p), B(-1, q, 3) and C(-3, 3, 0) are collinear, then find

a) the ratio in which point C divides the line segment AB

b) The values of p and q.

ii) Find the perpendicular distance of the point (1, 0, 0) from the line

$$\frac{x-1}{2} = \frac{y+1}{-3} = \frac{z+10}{8}$$

Also, find the co-ordinates of the foot of the perpendicular.

iii) Find the vector equation of the plane passing through the intersection of the planes

$\bar{r} \cdot (2\bar{i} + 2\bar{j} - 3\bar{k}) = 8$, $\bar{r} \cdot (2\bar{i} + 4\bar{j} + 3\bar{k}) = 7$

and through the point $(2, 1, 3)$.

Q3) B) Attempt any 2/3 (4M each) [8M]

i) Minimize $Z = 20x + 9y$ subject to

$x \geq 0, y \geq 0, \quad 2x + y \geq 36$ &
$6x + y \geq 60$

Also find the minimum value of Z.

ii) Find the shortest distance between the following pair of line

$\bar{r} = (\bar{i} + 2\bar{j} + \bar{k}) + \lambda(\bar{i} - \bar{j} + \bar{k})$ &

$\bar{r} = (2\bar{i} - \bar{j} - \bar{k}) + \mu(2\bar{i} + \bar{j} + 2\bar{k})$

iii) If θ is the acute angle between the line represented by equation $ax^2 + 2hxy + by^2 = 0$ then prove that $\tan\theta = \left| \frac{2\sqrt{h^2 - ab}}{a+b} \right|$. Also find the conditions for perpendicular and coincident line.