SECTION-I

Q.1] A) Multiple choice question (2M each) [6M]

i) The principal solution of the equation

$\cot x = \frac{-1}{\sqrt{3}}$ is

(a) $\frac{\pi}{6}$ (b) $\frac{5\pi}{6}$ (c) $\frac{\pi}{3}$ (d) $\frac{2\pi}{3}$

ii) If truth values of p, q and r are T, F and T then find the truth value of

$[(p \rightarrow q) \wedge \sim r]$

(a) T (b) F

iii) The general solution of the trigonometric equation $\tan^2\theta = 1$ is

(a) $\theta = n\pi \pm \frac{\pi}{3}$, $n \in Z$

(b) $\theta = n\pi \pm \frac{\pi}{6}$, $n \in Z$

(c) $\theta = n\pi \pm \frac{\pi}{4}$, $n \in Z$

(d) $\theta = n\pi$, $n \in Z$

B) Attempt any 3/5 (2M each) [6M]

i) Write the dual of the following statements

(1) $(p \vee q) \wedge F$

(2) Ravi or Vari went to Chennai.

ii) Find k, if sum of the slopes of the lines represented by $x^2 + kxy - 10y^2 = 0$ is thrice their product.

iii) If the vectors $2\bar{i} + 4\bar{j} + 16\bar{k}$ and $6\bar{i} - p\bar{j} + 48\bar{k}$ are collinear, then find the value of p.

iv) If a lines makes an angle α, β, γ with co-ordinate axes, then prove that

$$\cos 2\alpha + \cos 2\beta + \cos 2\gamma = -1$$

v) Find the angle between the planes $\bar{r} \cdot (\bar{i} - 2\bar{j} + 3\bar{k}) = 5$ and the line $\bar{r} \cdot (\bar{i} + \bar{j} - \bar{k}) = \lambda(\bar{i} - \bar{j} + \bar{k})$.

Q.2 A) Attempt any 2/3 (3M each) [6M]

i) Show that the angle between any two diagonals of a cube is $\cos^{-1}\left(\frac{1}{3}\right)$.

For Solution

ii) Using vector method prove that the altitudes of a triangle are concurrent.

iii) State whether the following statement patterns are tautologies, contradictions or contingencies

$[(p \vee \sim q) \vee (\sim p \wedge q)] \wedge r$.

Q2)B) Attempt any 2/3 (4M each) [8M]

i) The sum of three numbers is 6. If we multiply third number by 3 and add it to the second number we get 11. By adding first and third numbers we get a number which is double the second number. Find these three numbers using matrices.

ii) Find p and q if the equation $2x^2 + 8xy + py^2 + qx + 2y - 15 = 0$ represents a pair of parallel lines.

iii) Show that the lines $\bar{r} = (2\bar{j} - 3\bar{k}) + \lambda(\bar{i} + 2\bar{j} + 3\bar{k})$ and $\bar{r} = (2\bar{i} + 6\bar{j} + 3\bar{k}) + \mu(2\bar{i} + 3\bar{j} + 4\bar{k})$ are coplanar. Also find the equation of the plane containing them.

Q3) A) Attempt any 2/3 (3M each) [6M]

i) Prove that a homogeneous equation of degree two in x and y, represents a pair of lines passing through the origin if $h^2 - ab \geq 0$

ii) Show that lines $\frac{x+1}{-10} = \frac{y+3}{-1} = \frac{z-4}{1}$ and $\frac{-x-10}{1} = \frac{-y-1}{3} = \frac{1-z}{-4}$ intersect and find the coordinates of the point of intersection.

iii) A variable plane which remains at a constant distance $3p$ from the origin cut the co-ordinate axes at A, B, C. Show that the locus of the centroid of the triangle ABC is

$$\frac{1}{x^2} + \frac{1}{y^2} + \frac{1}{z^2} = \frac{1}{p^2}$$

Q3) B) Attempt any 2/3 (4M each) [8M]

i) Minimize $Z = x + 2y$

if $x + 2y \geq 50$

$2x - y \leq 0$

$2x + y \leq 100$, $x, y \geq 0$

ii) Find the general solution of

$$\sin 2x + \sin 4x + \sin 6x = 0$$

For Solution

iii) Show that angle between any two diagonals of a cube is $\cos^{-1}\left(\frac{1}{3}\right)$.